

Capacity Limits of Multiuser Multiantenna Cognitive Networks

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Abstract

Unlike point-to-point cognitive radio, where the constraint imposed by the primary rigidly curbs the secondary throughput, multiple secondary users have the potential to more efficiently harvest the spectrum and share it among themselves. This paper analyzes the sum throughput of a multiuser cognitive radio system with multi-antenna base stations, either in the uplink or downlink mode. The primary and secondary have N and n users, respectively, and their base stations have M and m antennas, respectively. We show that an *uplink secondary* throughput grows with $\frac{m}{N+1} \log n$ if the primary is a downlink system, and grows with $\frac{m}{M+1} \log n$ if the primary is an uplink system. These growth rates are shown to be optimal and can be obtained with a simple threshold-based user selection rule. Furthermore, we show that the secondary throughput can grow proportional to $\log n$ while simultaneously pushing the interference on the primary down to zero, asymptotically. Furthermore, we show that a *downlink secondary* throughput grows with $m \log \log n$ in the presence of either an uplink or downlink primary system. In addition, the interference on the primary can be made to go to zero asymptotically while the secondary throughput increases proportionally to $\log \log n$. Thus, unlike the point-to-point case, multiuser cognitive radios can achieve non-trivial sum throughput despite stringent primary interference constraints.

I. INTRODUCTION

Currently, the spectrum assigned to licensed (primary) users is heavily under-utilized [1]. Cognitive radio aims to improve the utilization of spectrum by allowing cognitive (secondary) users to access the same spectrum as primary users, as long as any performance degradation of the primary users is tolerable.

In general, secondary users can access the spectrum via methods known as overlay, interweave, and underlay [2]. In the overlay technique the secondary user not only transmits its own signal, but also acts as a relay to compensate for its interference on the primary user. The overlay method depends on

the secondary transmitter having access to primary's message [3].¹ In the interweave technique [4], the secondary user first senses spectrum holes and then transmits in the detected holes. Reliable sensing in the presence of fading and shadowing has proved to be challenging [5]. Finally, in the underlay technique [6], the secondary can transmit as long as the interference caused on the primary is less than a pre-defined threshold. The secondary user in this case is neither required to know the primary user's message nor restricted to transmit in spectrum holes.

This paper studies performance limits of an underlay cognitive network consisting of multi-user and multi-antenna primary and secondary systems. The primary and secondary systems are subject to mutual interference, where the secondary has to comply with a set of interference constraints imposed by the primary. We are interested in the average sum rate (throughput) of the secondary system as the number of secondary users grows. Moreover, we study how the secondary throughput is affected by the size of primary network as well as the severity of the interference constraints, which is one of the key issues in the design of an underlay cognitive network.

A summary of the results of this paper is as follows. We assume that the primary and secondary have N and n users, respectively, and their base stations have M and m antennas, respectively.

- **Secondary uplink (MAC):** the secondary average throughput is shown to grow as $\Theta(\log n)$, which is achieved by a threshold-based user selection rule. More precisely, the average throughput of the secondary MAC channel grows as $\frac{m}{N+1} \log n + O(1)$ when it coexists with the primary broadcast channel, and grows as $\frac{m}{M+1} \log n + O(1)$ when it coexists with the primary MAC channel. By developing asymptotically tight upper bounds, these growth rates are further proven to be optimal. Moreover, the interference on the primary system can be asymptotically forced to *zero*, while the secondary throughput still grows as $\Theta(\log n)$. Specifically, for some non-negative exponent q , the interference on the primary can be made to decline as $\Theta(n^{-q})$, while the throughput of a secondary MAC grows as $\frac{m-qN}{N+1} \log n + O(1)$ and $\frac{m-qM}{M+1} \log n + O(1)$, respectively in cases of primary broadcast and MAC channel. The above results imply that asymptotically the secondary system can attain a non-trivial throughput *without* degrading the performance of the primary system.
- **Secondary downlink (broadcast):** the secondary average throughput is shown to scale with $m \log \log n + O(1)$ in the presence of either the primary broadcast or MAC channel. Hence, the growth rate of throughput is unaffected (thus optimal) by the presence of the primary system. In addition, the interference on the primary can be asymptotically forced to *zero*, while maintaining the secondary

¹Sometimes, this is referred to as an interference channel with degraded message sets.

throughput as $\Theta(\log \log n)$. Specifically, for an arbitrary exponent $0 < q < 1$, the interference can be made to decline as $\Theta((\log n)^{-q})$, while the secondary average throughput grows as $m(1 - q) \log \log n + O(1)$.

Some of the related earlier work is as follows. Much of the past work in the underlay cognitive radio involves point-to-point primary and secondary systems. Ghasemi et al [6] studies the ergodic capacity of a point-to-point secondary link under various fading channels. Multiple antennas at the secondary transmitter are exploited by [7] to manage the tradeoff between the secondary throughput and the interference on the primary. In the context of multi-user cognitive radios, Zhang et al [8] studies the power allocation of a single-antenna secondary system under various transmit power constraints as well as interference constraints. Gastpar [9] studies the secondary capacity via translating a receive power constraint into a transmit power constraint.

Recently, ideas from opportunistic communication [10] were used in underlay cognitive radios by selectively activating one or more secondary users to maximize the secondary throughput while satisfying interference constraints. The user selection in cognitive radio is complicated because the secondary system must be mindful of two criteria: the interference on the primary and the rate provided to the secondary. Karama et al [11] selects secondary users with channels almost orthogonal to a single primary user, so that the interference on the primary is reduced. Jamal et al [12], [13] obtains interesting scaling results for the sum rate by selecting users causing the least interference. Some distinctions of our work and [12], [13] are worth noting. First, Jamal et al [12], [13] studies the hardening of sum rate via convergence in probability, while we analyze the average throughput, which requires a very different approach.² Second, we study a multi-antenna cognitive network whereas [12], [13] considers a single antenna network. Third, we study the effect of the primary network size (number of constraints) on the secondary throughput, while [12], [13] considers a single primary constraint.

²In general, convergence in probability does not imply convergence in any moment (thus average throughput) [14]. For example, consider a sequence of rates $R_n = \log(1 + X_n)$, where

$$X_n = \begin{cases} 1 & \text{with probability } 1 - \frac{1}{n} \\ \exp(n^2) & \text{with probability } \frac{1}{n} \end{cases}$$

Then, $\lim_{n \uparrow \infty} R_n = \log 2$ in probability, however, $\lim_{n \uparrow \infty} \mathbb{E}[R_n] = \infty$ in probability. Therefore, the average rate $\mathbb{E}[R_n]$ cannot be predicted based on the hardening (in probability) of R_n .

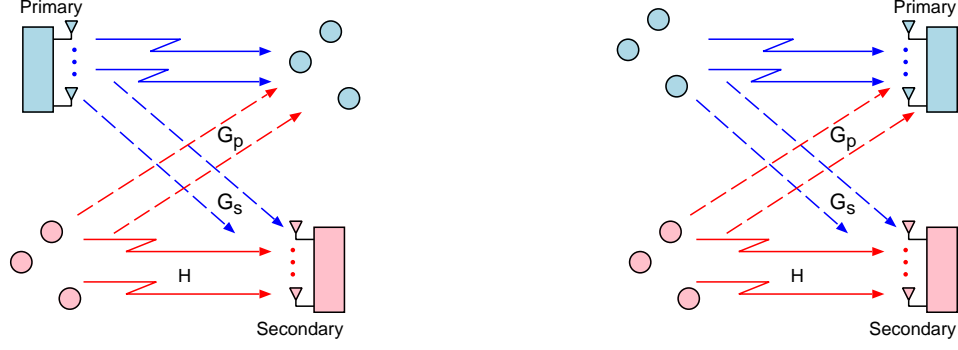


Fig. 1. Coexistence of the secondary MAC channel and the primary system

We use the following notation: $[\cdot]_{i,j}$ refers to the (i, j) element in a matrix, $|\cdot|$ refers to the cardinality of a set or the Euclidean norm of a vector, $\text{diag}(\cdot)$ refers to a diagonal matrix, $\text{tr}(\cdot)$ refers to the trace of a matrix, and $I_{k \times k}$ refers to the $k \times k$ identity matrix. All $\log(\cdot)$ is natural base. For any $\epsilon > 0$, some positive c_1 and c_2 , and sufficiently large n :

$$\begin{aligned} f(n) = O(g(n)) : & \quad |f(n)| < c_1 |g(n)| \\ f(n) = \Theta(g(n)) : & \quad c_2 |g(n)| < |f(n)| < c_1 |g(n)| \\ f(n) = o(g(n)) : & \quad |f(n)| < \epsilon |g(n)| \end{aligned}$$

We let $\mathcal{R}_{mac,w/o}^{opt}$ and $\mathcal{R}_{bc,w/o}^{opt}$ be the *maximum* average throughput achieved by the secondary MAC and broadcast channel *in the absence of* the primary, respectively. In this case, we have regular MAC and broadcast channels, and it is well known that $\mathcal{R}_{mac,w/o}^{opt}$ scales as $m \log n$, and $\mathcal{R}_{bc,w/o}^{opt}$ scales as $m \log \log n$.

The remainder of this paper is organized as follows. Section II describes the system model. The average throughput of the secondary MAC channel is studied in Section III, where in Section III-C we prove the achieved throughput is asymptotically optimal. The average throughput of the secondary broadcast channel is investigated in Section IV. Numerical results are shown in Section V. Finally, Section VI concludes this paper.

II. SYSTEM MODEL

We consider a cognitive network consisting of a primary and a secondary, each being either a MAC or broadcast channel (Figure 1 and Figure 2). The primary system has one base station with M antennas

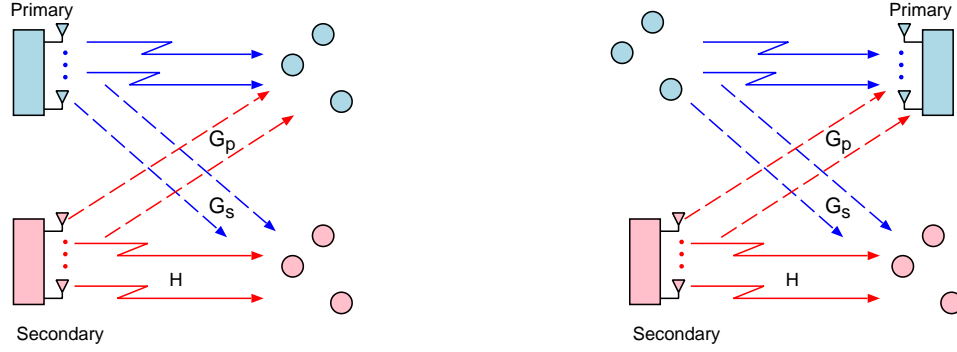


Fig. 2. Coexistence of the secondary broadcast channel and the primary system

and N users, while the secondary system consists of one base station with m antennas and n users. The primary and secondary are subject to mutual interference, which is treated as noise. The secondary system must comply with a set of interference power constraints imposed by the primary. For simplicity of exposition, at the beginning primary and secondary users (except base stations) are assumed to have one antenna, however, as shown in the sequel, most of the results can be directly extended to a scenario where each user has multiple antennas.

A block-fading channel model is assumed. All channel coefficients are fixed throughout each transmission block, and are independent, identically distributed (i.i.d.) circularly-symmetric-complex-Gaussian with zero mean and unit variance, denoted by $\mathcal{CN}(0, 1)$. The secondary base station acts as a scheduler: For each transmission block, a subset of the secondary users is selected to transmit to (or receive from) the secondary base station. We denote the collection of selected (active) secondary users as \mathcal{S} .

We begin by introducing a system model that applies to all four scenarios in Figures 1 and 2, thus simplifying notation in the remainder of the paper. The secondary received signal is given by:

$$\mathbf{y} = \mathbf{H}(\mathcal{S}) \mathbf{x}_s + \mathbf{G}_s \mathbf{x}_p + \mathbf{w} \quad (1)$$

where \mathbf{y} represents the received signal vector, either signals at a multi-antenna base station (uplink) or at different users (downlink). $\mathbf{H}(\mathcal{S})$ is the channel coefficient matrix between the active secondary users and their base station. \mathbf{G}_s represents the cross channel coefficient matrix from the primary transmitter(s) to the secondary receiver(s). The primary and secondary transmit signal vectors are \mathbf{x}_p and \mathbf{x}_s . The variable \mathbf{w} is the received noise vector, where each entry of \mathbf{w} is i.i.d. $\mathcal{CN}(0, 1)$.

We assume both primary and secondary systems use Gaussian signaling, subject to short-term power

constraints. The transmit covariance matrices of the primary and secondary systems are

$$Q_p = \mathbb{E}[\mathbf{x}_p \mathbf{x}_p^\dagger] \quad (2)$$

and

$$Q_s = \mathbb{E}[\mathbf{x}_s \mathbf{x}_s^\dagger] \quad (3)$$

When the secondary is a MAC channel, each secondary user is subject to an individual short term power constraint ρ_s . The users do not cooperate, therefore Q_s is diagonal:

$$Q_s = \text{diag}(\rho_1, \dots, \rho_{|\mathcal{S}|}) \quad (4)$$

where $\rho_\ell \leq \rho_s$, for $\ell = 1, \dots, |\mathcal{S}|$. In this case, $\mathbf{H}(\mathcal{S})$ has dimension $m \times |\mathcal{S}|$.

When the secondary is a broadcast channel, we assume the secondary base station is subject to a short term power constraint P_s :

$$\text{tr}(Q_s) \leq P_s \quad (5)$$

In this case, $\mathbf{H}(\mathcal{S})$ has dimension $|\mathcal{S}| \times m$.

When the primary is a MAC channel, each primary user transmits with power ρ_p without user cooperation:

$$Q_p = \rho_p I_{N \times N} \quad (6)$$

Furthermore, each receive antenna at the primary base station can tolerate interference with power Γ from the secondary system,³ that is

$$[\mathbf{G}_p Q_s \mathbf{G}_p^\dagger]_{\ell, \ell} \leq \Gamma \quad (7)$$

for $\ell = 1, \dots, M$, where \mathbf{G}_p represents the cross channel coefficient matrix from the secondary base station (or active users) to the primary base station.

When the primary is a broadcast channel, the power constraint at the primary base station is $\text{tr}(Q_p) \leq P_p$. For simplicity, we assume⁴

$$Q_p = \frac{P_p}{M} I_{M \times M} \quad (8)$$

Furthermore, each primary user tolerates interference with power Γ :

$$[\mathbf{G}_p Q_s \mathbf{G}_p^\dagger]_{\ell, \ell} \leq \Gamma \quad (9)$$

for $\ell = 1, \dots, N$, where \mathbf{G}_p is the cross channel coefficient matrix from the secondary base station (or active users) to the primary users.

³If each primary antenna or user tolerates a different interference power, the results of this paper still hold, as seen later.

⁴The asymptotic results remain the same, even if we allow Q_p to be an arbitrary covariance matrix.

III. COGNITIVE MAC CHANNEL

Consider a MAC secondary in the presence of either a broadcast or MAC primary. We wish to find how much throughput is available to the secondary subject to rigid constraints on the secondary-on-primary interference. We first construct a transmission strategy and find the corresponding (achievable) average throughput. Then, we develop upper bounds that are tight with respect to the throughput achieved.

The framework for the transmission strategy is as follows: For each transmission block, the secondary base station determines an active user set \mathcal{S} as well as transmit power for all active users Q_s . For each transmission, from (1), the sum rate (throughput) of the secondary system is:

$$R_{mac} = \log \det \left(I + \mathbf{H}(\mathcal{S}) Q_s \mathbf{H}^\dagger(\mathcal{S}) + \mathbf{G}_s Q_p \mathbf{G}_s^\dagger \right) - \log \det \left(I + \mathbf{G}_s Q_p \mathbf{G}_s^\dagger \right) \quad (10)$$

subject to the interference constraints (9) and (7) for the primary broadcast and MAC channel respectively.

The secondary average throughput is given by

$$\mathcal{R}_{mac} = \mathbb{E}[R_{mac}] \quad (11)$$

For the development of upper bounds, we assume the secondary base station knows all the channels. This is a genie-like argument that is used solely for development of upper bounds. For the achievable scheme, the requirement is more modest and is outlined after the description of the achievable scheme (see Remark 1).

A. Achievable Scheme

The objective is to choose \mathcal{S} and Q_s , i.e., the secondary active transmitters and their power, such that secondary throughput is maximized subject to interference constraints on the primary.

The choice of \mathcal{S} and Q_s is coupled through the interference constraints: either more secondary users can transmit with smaller power, or fewer of them with higher power. We focus on a simple power policy: All active secondary users transmit with the maximum allowed power ρ_s . Hence, given an active user set \mathcal{S} , we have

$$Q_s = \rho_s I_{|\mathcal{S}| \times |\mathcal{S}|} \quad (12)$$

It will be shown that the on-off transmission (without any further power adaptation) suffices to (asymptotically) achieve the maximum average throughput. Furthermore, its simplicity facilitates analysis.

Recall that each primary user can tolerate interference with power Γ . The interference on a primary user is guaranteed to be below this level if k_s secondary users are active, each causing interference no more than $\alpha = \frac{\Gamma}{k_s}$. This bound allows us to honor the interference constraints on the primary while decoupling

the action of different secondary users. Based on this observation, we construct a user selection rule as follows. First, we define an eligible secondary user set that disqualifies users that cause too much interference on the primary:

$$\mathcal{A} = \begin{cases} \{i : \rho_s |[\mathbf{G}_p]_{ji}|^2 < \alpha, \text{ for } j = 1, \dots, N\} & \text{primary broadcast} \\ \{i : \rho_s |[\mathbf{G}_p]_{ji}|^2 < \alpha, \text{ for } j = 1, \dots, M\} & \text{primary MAC} \end{cases} \quad (13)$$

where $[\mathbf{G}_p]_{ji}$ is the channel coefficient from the secondary user i to the primary user (antenna) j , and α is a pre-designed interference quota. A secondary user is eligible if its interference on each primary user (antenna) is less than α . Now, to satisfy the interference bound, we limit the number of secondary transmitters to no more than k_s , where

$$k_s = \frac{\Gamma}{\alpha} \quad (14)$$

If $|\mathcal{A}| \leq k_s$, then all eligible users can transmit. If $|\mathcal{A}| > k_s$, then k_s users will be chosen *randomly* from among the eligible users to transmit.⁵ The number of eligible users, $|\mathcal{A}|$, is a random variable; the number of active users is

$$|\mathcal{S}| = \min(k_s, |\mathcal{A}|) \quad (15)$$

The transmission of $|\mathcal{S}|$ eligible users induces interference no more than Γ on any primary user or antenna. Notice that the manner of user selection guarantees that the channel coefficients in $\mathbf{H}(\mathcal{S})$ remain independent and distributed as $\mathcal{CN}(0, 1)$.

Now we want to design an interference quota α to maximize the secondary average throughput. Neither very small nor very large values of α are useful within our framework: If α is very small, for most transmissions few (if any) secondary users will be eligible, thus the secondary throughput will be small. If $\alpha > \Gamma$, any transmitting user might violate the interference constraint, so the secondary must shut down (equivalently, we have $k_s < 1$). The value of individual interference constraint α , or equivalently k_s , must be set somewhere between these extremes.

Clearly, a desirable outcome would be to allow exactly the number of users that are indeed eligible for transmission, i.e., $k_s \approx |\mathcal{A}|$. But one cannot guarantee this in advance, because $|\mathcal{A}|$ is a random variable. Motivated by this general insight, we choose α such that

$$k_s = \mathbb{E}[|\mathcal{A}|] \quad (16)$$

⁵Naturally the number of active users must be an integer, i.e., $\lfloor k_s \rfloor$. We do not carry the floor operation in the following developments for simplicity, noting that due to the asymptotic nature of the analysis, the floor operation has no effect on the final results.

In Section III-C, we will verify that this choice of α is enough to asymptotically achieve the maximum throughput.

Remark 1: The above scheme does not require the secondary users to have full channel knowledge. Each secondary user can compare its own cross channel gains with a pre-defined interference quota α , and then decide its eligibility. After this, each eligible user can inform the secondary base station via 1-bit, so that the secondary base station can determine \mathcal{A} without knowing the cross channels from the secondary users to the primary system. The secondary channels $\mathbf{H}(\mathcal{S})$ and the cross channels \mathbf{G}_s can be estimated at the secondary base station. Therefore, this scheme can be implemented with little exchange of channel knowledge.

B. Throughput Calculation

1) *Secondary MAC with Primary Broadcast:* The primary base station transmits to N primary users, where each user tolerates interference with power Γ . Notice that in (13), $[\mathbf{G}_p]_{ji}$ is the channel coefficient from the secondary user i to the primary user j which is i.i.d. $\mathcal{CN}(0, 1)$. Thus, $|[\mathbf{G}_p]_{ji}|^2$ is i.i.d. exponential. Therefore, $|\mathcal{A}|$ is binomially distributed with parameter (n, p) , where

$$p = \left(1 - e^{-\frac{\alpha}{\rho_s}}\right)^N \quad (17)$$

For small $\frac{\alpha}{\rho_s}$, we have

$$p \approx \left(\frac{\alpha}{\rho_s}\right)^N \quad (18)$$

From (16), the interference quota α is chosen such that

$$k_s = np \approx n \left(\frac{\alpha}{\rho_s}\right)^N \quad (19)$$

Substitute $\alpha = \frac{\Gamma}{k_s}$ into the above equation, and denote the associated solution for k_s as \bar{k}_s :

$$\bar{k}_s = \left(\frac{\Gamma}{\rho_s}\right)^{\frac{N}{N+1}} (n)^{\frac{1}{N+1}} \quad (20)$$

Thus, we can see $\Theta(n^{\frac{1}{N+1}})$ secondary users are allowed to transmit, and the interference quota is on the order of $\Theta(n^{-\frac{1}{N+1}})$. With the above choice of interference quota, or the number of allowable active users, we state one of the main results of this paper as follows.

Theorem 1: Consider a secondary MAC with a m -antenna base station and n users each with power constraint ρ_s . The secondary MAC operates in the presence of a primary broadcast channel transmitting with power P_p to N users each with interference tolerance Γ . The secondary average throughput satisfies:

$$\mathcal{R}_{mac} \geq \frac{m}{N+1} \log n + \frac{1}{N+1} \log (\rho_s \Gamma^N) - m \log(1 + P_p) + O(n^{-\frac{1}{N+1}} \log n) \quad (21)$$

$$\mathcal{R}_{mac} \leq \frac{m}{N+1} \log n + \frac{1}{N+1} \log (\rho_s \Gamma^N) - \mathcal{R}_I + O(n^{-\frac{1}{N+1}}) \quad (22)$$

with

$$\mathcal{R}_I = m_{\min} \log \left(1 + \frac{P_p}{M} \exp \left(\frac{1}{m_{\min}} \sum_{j=1}^{m_{\min}} \sum_{i=1}^{m_{\max}-j} \frac{1}{i} - \gamma \right) \right) \quad (23)$$

where $m_{\min} = \min(m, M)$ and $m_{\max} = \max(m, M)$. This throughput is achieved under the threshold-based user selection with the choice of \bar{k}_s given by (20).

Proof: See Appendix A. □

Remark 2: The essence of the above result is that the secondary average throughput grows as $\frac{m}{N+1} \log n + O(1)$, i.e., inversely proportional to the number of primary users. A noteworthy special case is when the primary base station chooses to transmit to a number of users equal to the number of its transmit antennas ($N = M$), a strategy which is known to be near-optimum in terms of sum-rate [15]. Under this condition:

$$\mathcal{R}_{mac} = \frac{m}{M+1} \log n + O(1)$$

Therefore, we have

$$\lim_{n \rightarrow \infty} \frac{\mathcal{R}_{mac}}{\mathcal{R}_{mac,w/o}^{opt}} = \frac{1}{M+1} \quad (24)$$

where $\mathcal{R}_{mac,w/o}^{opt}$ is the maximum average throughput of the secondary MAC *in the absence of* the primary system. This ratio shows that the *compliance penalty* of the secondary MAC system and its relationship with the characteristics of the primary network.

Remark 3: The results in Theorem 1 can be directly extended to a scenario where each primary user tolerates a different level of interference. As long as all primary users allow non-zero interference (no matter how small), we can let Γ be the minimum allowable interference, and the theorem still holds.

So far we have analyzed the effect of small but constant primary interference constraints, and shown that the secondary throughput improves with increasing the number of secondary users. However, the flexibility provided by the increasing number of secondary users can be exploited not only to increase secondary throughput, but also to reduce the primary interference. In fact, it is possible to simultaneously suppress the interference on the primary down to *zero* while increasing the secondary throughput proportional to $\log n$. The following corollary makes this idea precise:

Corollary 1: Assuming the interference on each primary user is bounded as $\Theta(n^{-q})$, the average secondary throughput satisfies

$$\mathcal{R}_{mac} = \frac{m - qN}{N + 1} \log n + O(1) \quad (25)$$

where $0 < q < \frac{m}{N}$.

Proof: Because the proof of Theorem 1 holds for $\Gamma = \Theta(n^{-q})$, the corollary follows by substituting $\Gamma = \Theta(n^{-q})$ into the lower and upper bounds given by Theorem 1. \square

Remark 4: The corollary above explores a tradeoff where primary interference is made to decrease polynomially, i.e., proportional to n^{-q} . We saw that this leads to a secondary sum rate that decreases linearly in q . If we reduce the primary interference more slowly, i.e., decreasing as $\Theta(\frac{1}{\log n})$, the growth rate of secondary sum-rate will behave as though the primary interference constraint is fixed. Conversely, if we try to suppress the primary interference faster than $\Theta(n^{-q})$, the secondary throughput will asymptotically remain stagnant or will go to zero.

2) *Secondary MAC with a Primary MAC:* Recall that each antenna at the primary base station allows interference with power Γ . By regarding each antenna of the primary base station as a virtual user, we can re-use most of the analysis that was developed in the previous section. Thus, the steps leading to Eq. (20) can be repeated to obtain the number of allowable active secondary users:

$$\bar{k}_s = \left(\frac{\Gamma}{\rho_s} \right)^{\frac{M}{M+1}} (n)^{\frac{1}{M+1}} \quad (26)$$

With this allowable active users \bar{k}_s and slight modifications, we obtain a result that parallels Theorem 1.

Theorem 2: Consider a secondary MAC with a m -antenna base station and n users each with power constraint ρ_s . The secondary MAC operates in the presence of a primary MAC channel where each user transmits with power ρ_p to a M -antenna base station with interference tolerance Γ on each antenna. The secondary average throughput satisfies:

$$\mathcal{R}_{mac} \geq \frac{m}{M+1} \log n + \frac{1}{M+1} \log(\rho_s \Gamma^M) - m \log(1 + \rho_p N) + O(n^{-\frac{1}{M+1}} \log n) \quad (27)$$

$$\mathcal{R}_{mac} \leq \frac{m}{M+1} \log n + \frac{1}{M+1} \log(\rho_s \Gamma^M) - \mathcal{R}_I + O(n^{-\frac{1}{M+1}}) \quad (28)$$

with

$$\mathcal{R}_I = m_{\min} \log \left(1 + \rho_p \exp \left(\frac{1}{m_{\min}} \sum_{j=1}^{m_{\min}} \sum_{i=1}^{m_{\max}-j} \frac{1}{i} - \gamma \right) \right) \quad (29)$$

where $m_{\min} = \min(m, N)$ and $m_{\max} = \max(m, N)$. This throughput is achieved under the threshold-based user selection with the choice of \bar{k}_s given by (26).

A tradeoff exists between the primary interference reduction and the secondary throughput enhancement, which is stated by the following corollary. All the remarks made after Corollary 1 are applicable here.

Corollary 2: Assuming the interference on each antenna of the primary base station is bounded as $\Theta(n^{-q})$, the average secondary throughput satisfies

$$\mathcal{R}_{mac} = \frac{m - qM}{M + 1} \log n + O(1) \quad (30)$$

where $0 < q < \frac{m}{M}$.

C. Upper Bounds for Secondary Throughput

So far we have seen achievable rates of a cognitive MAC channel in the presence of either a primary broadcast or MAC. We now develop corresponding upper bounds.

Theorem 3: Consider a secondary MAC with a m -antenna base station and n users. The *maximum* average throughput of the secondary, \mathcal{R}_{mac}^{opt} , satisfies

$$\mathcal{R}_{mac}^{opt} \leq \frac{m}{N + 1} \log n + O(\log \log n) \quad (31)$$

in the presence of a primary broadcast channel transmitting to N users. Similarly, \mathcal{R}_{mac}^{opt} satisfies

$$\mathcal{R}_{mac}^{opt} \leq \frac{m}{M + 1} \log n + O(\log \log n) \quad (32)$$

in the presence of a primary MAC, where each user transmits to a M -antenna base station.

Proof: See Appendix B. □

Remark 5: By comparing the upper bounds with the achievable rates obtained by the thresholding strategy, we see that the achievable rates are at most $O(\log \log n)$ away from the upper bounds, a difference which is negligible relative to the dominant term $\Theta(\log n)$. Thus, the growth of the *maximum* average throughput of a cognitive MAC is $\frac{m}{N+1} \log n$ in the presence of the primary broadcast channel, and $\frac{m}{M+1} \log n$ in the presence of the primary MAC channel. Both the achievable rates and the upper bounds show that the average cognitive sum-rate is inversely proportional to the number of primary-imposed constraints, asymptotically.

D. Discussion

Recall that our method determines eligible cognitive MAC users based on their cross channel gains. To satisfy the interference constraints, our selection rule then allows $\Theta(n^{\frac{1}{N+1}})$, or $\Theta(n^{\frac{1}{M+1}})$, of these

users to be active simultaneously, in the presence of either the primary broadcast or MAC. If there are more eligible users than the allowed number, we choose from among the eligible users randomly. In this process, the forward channel gain of the cognitive users does not come into play, and still an optimal growth rate is achieved. This can be intuitively explained as follows. The total received signal power at the cognitive base station grows linearly with the number of active users, and the total received signal power determines the sum rate. On the other hand, selecting good cognitive users according to their secondary channel strengths can only offer logarithmic power gains (with respect to n) [10], which is negligible compared to the linear gains due to increasing the number of active users. Therefore the cross channel gains are more important in this case.⁶ Note that we do not imply that knowledge of the cognitive forward channel is useless; our conclusion only says that once the cross channels are taken into account, the *asymptotic growth* of the secondary throughput cannot be improved by any use of the cognitive forward channel.

Although we have allowed the base stations to have multiple antennas, so far the users have been assumed to have only one antenna. We now consider a generalization to the case where all users have multiple antennas. Consider a secondary MAC in the presence of a primary broadcast, where each primary and secondary user have t_p and t_s antennas respectively. We apply a separate interference constraint on each antenna of each primary user, which guarantees the satisfaction of the overall interference constraint on any primary user. On each of the t_s -antenna secondary users, we shall allocate $t_s - 1$ degrees of freedom for zero-forcing and only one degree of freedom for cognitive transmission. Using this strategy, we can ensure that $t_s - 1$ of the receive antennas on the primary are exempt from interference. Thus, the total number of interference constraints will reduce from $t_p N$ to $t_p N + 1 - t_s$. By using an analysis similar to the development of Theorem 1, one can show that the growth rate $\frac{m \log n}{\max(1, t_p N + 2 - t_s)}$ is achievable. For the converse, the situation is more complicated, because here the correlation among the antennas of the secondary users must be accounted for. Nevertheless, in some cases it is possible to show without much difficulty that the above achieved throughput is indeed asymptotically optimal. For example, in the presence of the primary MAC, if $t_s > M$, the secondary MAC channel can have a throughput that grows as $m \log n$ by letting each active secondary user completely eliminate the interference on the primary. Similarly, in the presence of a primary broadcast channel, if $t_s > t_p N$, the secondary MAC channel can also have a throughput that grows as $m \log n$. The achieved growth rate is optimal because it coincides

⁶In a somewhat different context, the work of Jamal et al. [13] also indicates that cross channels can be more important than the forward channels.

with the the growth rate of $\mathcal{R}_{mac,w/o}^{opt}$, which is always an upper bound.

IV. COGNITIVE BROADCAST CHANNEL

A. Achievable Scheme

We consider a random beam-forming technique where the secondary base station opportunistically transmits to m secondary users simultaneously [16]. Specifically, the secondary base station constructs m orthonormal beams, denoted by $\{\phi_j\}_{j=1}^m$, and assigns each beam to a secondary user. Then, the secondary base station broadcasts to m selected users. The selection of users and beam assignment will be addressed shortly.

Considering an equal power allocation among m users, the transmitted signal from the secondary base station is given by:

$$\mathbf{x}_s = \sum_{j=1}^m \sqrt{\frac{P}{m}} \phi_j x_j \quad (33)$$

where ϕ_j is the beam-forming vector j with dimension $m \times 1$, x_j is the signal transmitted along the beam j , and P is the total transmit power. In this case, we have

$$Q_s = \frac{P}{m} I_{m \times m} \quad (34)$$

Notice that P is subject to the power constraint P_s as well as a set of interference constraints imposed by the primary. Thus, the value of P depends on the cross channels from the secondary base station to the primary system.

Assuming the beam j is assigned to user i . From (1) and (33), the received signal at the secondary user i is given by

$$y_i = \mathbf{h}_i^\dagger \phi_j x_j + \sum_{k \neq j} \mathbf{h}_i^\dagger \phi_k x_k + \mathbf{g}_{s,i}^\dagger \mathbf{x}_p + w_i \quad (35)$$

where \mathbf{h}_i^\dagger is the $1 \times m$ vector of channel coefficient from the secondary base station to the secondary user i , and $\mathbf{g}_{s,i}^\dagger$ is the $1 \times M$ (or $1 \times N$) vector of channel coefficients from the primary base station (or users) to the secondary user i . The received signal-to-noise-plus-interference-ratio (SINR) at the secondary user i (with respect to beam j) is

$$\text{SINR}_{i,j} = \frac{\frac{P}{m} |\mathbf{h}_i^\dagger \phi_j|^2}{1 + \frac{P}{m} \sum_{k \neq j} |\mathbf{h}_i^\dagger \phi_k|^2 + \mathbf{g}_{s,i}^\dagger Q_p \mathbf{g}_{s,i}} \quad (36)$$

The random beam technique assigns each beam to the secondary user that results in the highest SINR. Because the probability of more than two beams being assigned to the same secondary user is negligible,

we have [16]

$$\mathcal{R}_{bc} \approx \mathbb{E} \left[\sum_{j=1}^m \log \left(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,j} \right) \right] \quad (37)$$

$$= m \mathbb{E} \left[\log \left(1 + \max_{1 \leq i \leq n} \text{SINR}_{i,j} \right) \right] \quad (38)$$

The above analysis holds in the presence of either the primary broadcast or MAC channel; the only difference is the constraints on P and Q_p . Since the SINR is symmetric across all beams, the subscript j will be omitted in the following analysis.

Remark 6: We briefly address the issue of channel state information. All users are assumed to have receiver side channel state information. On the transmit side, the secondary base station does not need to have full channel knowledge; only the SINR is needed. Each secondary user can estimate its own SINR with respect to each beam, and feed it back to the secondary base station [16]. Based on collected SINR, the secondary base station performs user selection. The secondary base station needs to know \mathbf{G}_p to adjust P such that the interference constraints on the primary are satisfied.

B. Throughput Calculation

1) *Secondary Broadcast with Primary Broadcast:* The secondary system has to comply with the constraints on N primary users. To maximize the throughput, the secondary base station transmits at the maximum allowable power. From (9) and (34), we have

$$P = \min \left(\frac{m\Gamma}{|\mathbf{g}_{p,1}^\dagger|^2}, \dots, \frac{m\Gamma}{|\mathbf{g}_{p,N}^\dagger|^2}, P_s \right) \quad (39)$$

where $\mathbf{g}_{p,\ell}^\dagger$ is the row ℓ of \mathbf{G}_p . Then, we substitute Q_p given by (8) into (36), and obtain the SINR at the secondary user i with respect to the beam j :

$$\text{SINR}_i = \frac{|\mathbf{h}_i^\dagger \phi_j|^2}{\frac{m}{P} + \sum_{k \neq j} |\mathbf{h}_i^\dagger \phi_k|^2 + \frac{mP_p}{MP} |\mathbf{g}_{s,i}|^2} \quad (40)$$

Our analysis of $\max_i \text{SINR}_i$, which is required to evaluate the throughput in Eq. (38), does not follow [16] because the denominator involves a sum of two Gamma distributions with different scale parameters: $\sum_{k \neq j} |\mathbf{h}_i^\dagger \phi_k|^2$ has $\text{Gamma}(m-1, 1)$ and $\frac{mP_p}{MP} |\mathbf{g}_{s,i}|^2$ has $\text{Gamma}(M, \frac{mP_p}{MP})$. Fortunately, lower and upper bounds can be leveraged to simplify the analysis. We define:

$$\theta = \frac{mP_p}{MP} \quad (41)$$

We consider the case when $\frac{mP_p}{MP_s} \geq 1$. The techniques can then be generalized to the case of $\frac{mP_p}{MP_s} < 1$.⁷

⁷When $\frac{mP_p}{MP_s} < 1$, one can define $\theta = \max(\frac{mP_p}{MP}, 1)$. Then, we can use Bayesian expansion via conditioning on $\{P < \frac{mP_p}{M}\}$ and its complement, where both conditional terms can be shown to have the same growth rate.

When $\frac{mP_p}{MP_s} \geq 1$, we have $\theta \geq 1$ for all P . We define:

$$L_i = \frac{|\mathbf{h}_i^\dagger \phi_j|^2}{\frac{m}{P} + \theta(\sum_{k \neq j} |\mathbf{h}_i^\dagger \phi_k|^2 + |\mathbf{g}_{s,i}|^2)} \quad (42)$$

and

$$U_i = \frac{|\mathbf{h}_i^\dagger \phi_j|^2}{\frac{m}{P} + \theta|\mathbf{g}_{s,i}|^2} \quad (43)$$

where L_i and U_i are random variables that depend on channel realizations. Conditioned on P , the denominators of L_i and U_i have Gamma distributions, which simplifies the analysis.

For $1 \leq i \leq n$, we have

$$L_i \leq \text{SINR}_i \leq U_i \quad (44)$$

Hence,

$$L_{\max} \leq \max_{1 \leq i \leq n} \text{SINR}_i \leq U_{\max} \quad (45)$$

where $L_{\max} = \max_i L_i$ and $U_{\max} = \max_i U_i$. Therefore for any x , we have

$$\mathbb{P}(L_{\max} > x) \leq \mathbb{P}(\max_{1 \leq i \leq n} \text{SINR}_i > x) \leq \mathbb{P}(U_{\max} > x) \quad (46)$$

which implies [17] that $\max_i \text{SINR}_i$ is stochastically greater than L_{\max} , but stochastically smaller than U_{\max} . We now use the following fact about stochastic ordering:

Lemma 1 ([17]): If random variable X is stochastically smaller than Y and $h(\cdot)$ is an increasing function, assuming $h(X)$ and $h(Y)$ are measurable according to their distributions:

$$\mathbb{E}[h(X)] \leq \mathbb{E}[h(Y)] \quad (47)$$

Based on the above lemma, the secondary average throughput is bounded as follows:

$$m\mathbb{E}[\log(1 + L_{\max})] \leq \mathcal{R}_{bc} \leq m\mathbb{E}[\log(1 + U_{\max})] \quad (48)$$

We study the lower and upper bounds given by (48), instead of directly analyzing \mathcal{R}_{bc} . Some useful properties of L_{\max} and U_{\max} are as follows.

Lemma 2: Conditioned on $P = \rho$,

$$\mathbb{P}\left(L_{\max} \geq b_n - \frac{\rho}{m} \log \log n \mid P = \rho\right) = 1 - \Theta\left(\frac{1}{n}\right) \quad (49)$$

$$\mathbb{P}\left(U_{\max} < d_n + \frac{\rho}{m} \log \log n \mid P = \rho\right) = 1 - \Theta\left(\frac{1}{\log n}\right) \quad (50)$$

$$\mathbb{E}\left[U_{\max} \mid U_{\max} > d_n + \frac{\rho}{m} \log \log n, P = \rho\right] < O(n \log n) \quad (51)$$

where $b_n = \frac{\rho}{m} \log n - \frac{\rho(m+M-1)}{m} \log \log n + O(\log \log \log n)$ and $d_n = \frac{\rho}{m} \log n - \frac{\rho M}{m} \log \log n + O(\log \log \log n)$.

Proof: See Appendix C. □

Based on the above two lemmas, we obtain the following results for the secondary throughput:

Theorem 4: Consider a secondary broadcast channel with n users and a m -antenna base station with power constraint P_s . The secondary broadcast operates in the presence of a primary broadcast channel transmitting with power P_p to N users each with interference tolerance Γ . The secondary average throughput satisfies:

$$\mathcal{R}_{bc} > m \log(\Gamma \log n) - m \log\left(\tilde{\mu}_1 + \frac{m\Gamma}{P_s}\right) + O\left(\frac{\log \log n}{\log n}\right)$$

$$\mathcal{R}_{bc} < m \log(\Gamma \log n) - m \log \tilde{\mu}_2 + O(1)$$

where $\tilde{\mu}_1 = \mathbb{E}[\max_{1 \leq i \leq N} |\mathbf{g}_{p,i}^\dagger|^2]$ and $\tilde{\mu}_2 = (\mathbb{E}[1/\max_{1 \leq i \leq N} |\mathbf{g}_{p,i}^\dagger|^2])^{-1}$.

Proof: See Appendix D. □

Remark 7: The result above states that $\mathcal{R}_{bc} = m \log \log n + O(1)$, thus

$$\lim_{n \rightarrow \infty} \frac{\mathcal{R}_{bc}}{\mathcal{R}_{bc,w/o}^{opt}} = 1 \quad (52)$$

where $\mathcal{R}_{bc,w/o}^{opt}$ is the maximum average throughput of the secondary broadcast channel *in the absence of* the primary system. Therefore, the achieved average throughput is *asymptotically optimal*, because we always have $\mathcal{R}_{bc} \leq \mathcal{R}_{bc,w/o}^{opt}$. Thus, we have a positive result: The growth rate of the secondary average throughput is unaffected by the constraints and interference imposed by the primary, as long as each primary user tolerates some small but fixed interference.

The above results naturally lead to the question: How small can we make the interference on the primary, while still having a secondary average throughput that grows as $\Theta(\log \log n)$. We find that Γ , the interference on each primary user, can asymptotically go to *zero*, as shown by the next corollary.

Corollary 3: Assuming the interference on each primary user is bounded as $\Theta((\log n)^{-q})$, the average secondary throughput satisfies:

$$\mathcal{R}_{bc} = (1 - q)m \log \log n + O(1) \quad (53)$$

where $0 < q < 1$.

Remark 8: Reducing the interference on the order of $\Theta((\log n)^{-q})$ sheds lights on how fast the interference can be reduced on the primary, while having a non-trivial secondary throughput. For $q > 1$,

it does not imply \mathcal{R}_{bc} is zero or negative; it only means that \mathcal{R}_{bc} is on the order of $o(\log \log n)$. Slower interference reduction, e.g. proportional to $\Theta((\log \log n)^{-1})$, will give maximal asymptotic growth of secondary throughput, i.e., $m \log \log n$.

2) *Secondary Broadcast with Primary MAC*: The analysis of this case closely parallels the analysis of the primary broadcast. The secondary transmit power is given by

$$P = \min \left(\frac{m\Gamma}{|\mathbf{g}_{p,1}^\dagger|^2}, \dots, \frac{m\Gamma}{|\mathbf{g}_{p,M}^\dagger|^2}, P_s \right) \quad (54)$$

where $\mathbf{g}_{p,\ell}^\dagger$ is the row ℓ of \mathbf{G}_p . The MAC primary system produces power $N\rho_p$ and has M interference constraints. From the viewpoint of the secondary, this is all the information that is needed. Therefore the analysis of Theorem 4 can be essentially repeated to obtain the following result.

Theorem 5: Consider a secondary broadcast channel with n users and a m -antenna base station with power constraint P_s . The secondary broadcast operates in the presence of a primary MAC where each user transmits with power ρ_p to a M -antenna base station with interference tolerance Γ on each antenna. The secondary average throughput satisfies:

$$\begin{aligned} \mathcal{R}_{bc} &> m \log(\Gamma \log n) - m \log\left(\tilde{\mu}_3 + \frac{m\Gamma}{P_s}\right) + O\left(\frac{\log \log n}{\log n}\right) \\ \mathcal{R}_{bc} &< m \log(\Gamma \log n) - m \log \tilde{\mu}_4 + O(1) \end{aligned}$$

where $\tilde{\mu}_3 = \mathbb{E}[\max_{1 \leq i \leq M} |\mathbf{g}_{p,i}^\dagger|^2]$ and $\tilde{\mu}_4 = (\mathbb{E}[1/\max_{1 \leq i \leq M} |\mathbf{g}_{p,i}^\dagger|^2])^{-1}$.

Remark 9: Theorem 4 and Theorem 5 can be extended to a scenario where each primary and secondary user has multiple antennas. A straightforward way is to regard each primary and secondary antenna as a virtual user. Using an analysis similar to the single-antenna case, the secondary broadcast channel can be shown to achieve a throughput scaling as $m \log \log n$ (thus optimal). The details are straight forward and are therefore omitted for brevity.

Similar to Corollary 3, we can also obtain the tradeoff between the primary interference reduction and the secondary throughput enhancement as follows. All the remarks following Corollary 3 apply to the present case as well.

Corollary 4: Assuming the interference on each antenna of the primary base station is bounded as $\Theta((\log n)^{-q})$, the average secondary throughput satisfies:

$$\mathcal{R}_{bc} = (1 - q)m \log \log n + O(1) \quad (55)$$

where $0 < q < 1$.

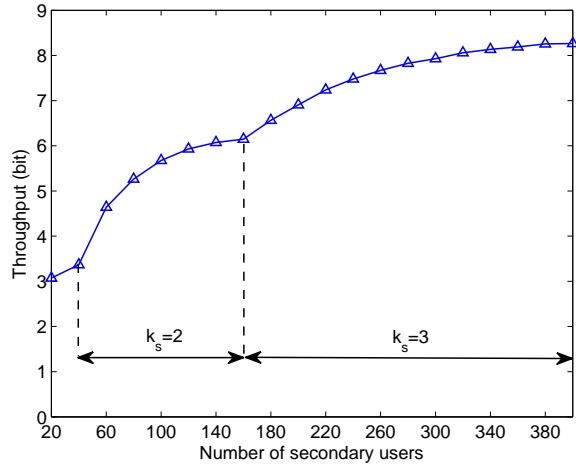


Fig. 3. Secondary MAC: Throughput versus user number ($\Gamma = 2$)

V. NUMERICAL RESULTS

In this section, we concentrate on numerical results in the presence of the primary broadcast channel; the results in the presence of the primary MAC channel are similar thus omitted. For all simulations, we consider: $P_p = P_s = \rho_s = 5$, the secondary base station has $m = 4$ antennas, and the primary base station has $M = 2$ antennas and the number of primary users is $N = 2$.

Figure 3 illustrates the secondary average throughput given by Theorem 1. The allowable interference power on each primary user is $\Gamma = 2$. The slope of the throughput curve is discontinuous at some points, because the allowable number of active secondary users must be an integer $\lfloor k_s \rfloor$ (also see Eq.(19)). As mentioned earlier, the floor operation does not affect the asymptotic results. Figure 4 presents the tradeoff between the tightness of the primary constraints and the secondary throughput, as shown by Corollary 1. The interference power constraint Γ is $2n^{-q}$ for $q = 0.1$ and 0.2 respectively. As expected, for $q = 0.2$ the interference on primary decreases faster than $q = 0.1$ and the secondary throughput increases more slowly.

Figure 5 shows the secondary throughput versus the number of secondary users in the presence of the primary broadcast channel (Theorem 4), where the interference power is $\Gamma = 2$. In Figure 6, we show the tradeoff between the secondary throughput and the interference on the primary, as described in Corollary 3. We set Γ to decline as $2(\log n)^{-q}$, for $q = 0.5$ and $q = 0.8$, respectively. Clearly, for $q = 0.5$, the interference power decreases faster than $q = 0.8$, while the secondary throughput increases more slowly.

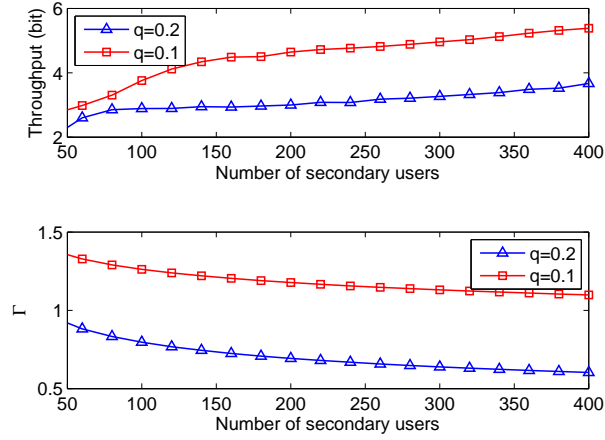


Fig. 4. Secondary MAC: Throughput versus user number ($\Gamma = 2n^{-q}$)

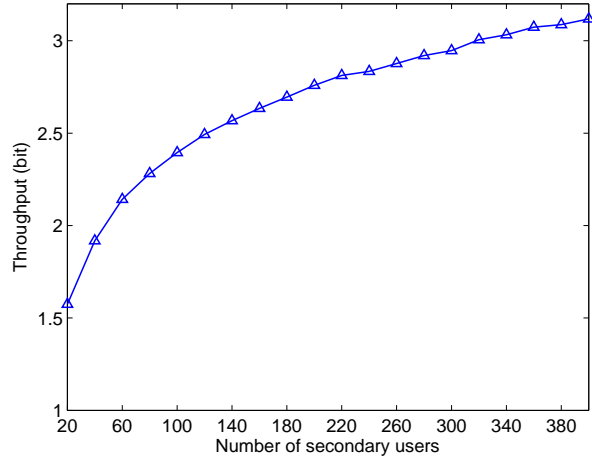


Fig. 5. Secondary broadcast: Throughput versus user number ($\Gamma = 2$)

VI. CONCLUSION

In this paper, we study the performance limits of an underlay cognitive network consisting of a multi-user and multi-antenna primary and secondary systems. We find the average throughput limits of the secondary system as well as the tradeoff between this throughput and the tightness of constraints imposed by the primary system. Given a set of interference power constraints on the primary, the maximum average throughput of the secondary MAC grows as $\frac{m}{N+1} \log n$ (primary MAC), and $\frac{m}{M+1} \log n$ (primary broadcast). These growth rates are attained by the simple threshold-based user selection rule. Interestingly,

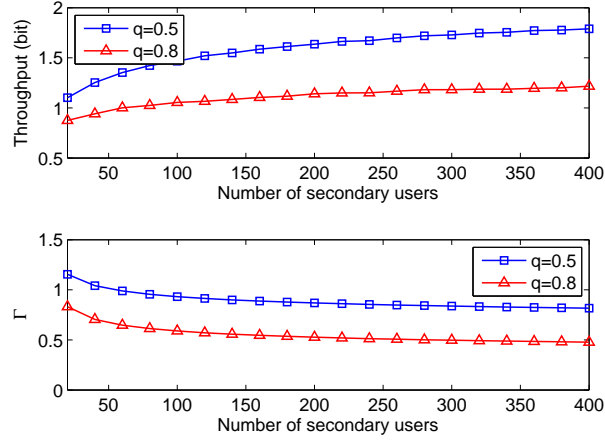


Fig. 6. Secondary broadcast: Throughput versus user number ($\Gamma = 2(\log n)^{-q}$)

the secondary system can force its interference on the primary to zero while maintaining a growth rate of $\Theta(\log n)$. For the secondary broadcast channel, the secondary average throughput can grow as $m \log \log n$ in the presence of either the primary broadcast or MAC channel. Hence, the growth rate of the throughput is unaffected by the presence of the primary (thus optimal). Furthermore, the interference on the primary can also be made to decline to zero, while maintaining the secondary average throughput to grow as $\Theta(\log \log n)$.

APPENDIX A

PROOF OF THEOREM 1

Proof: We rewrite (10) as

$$R_{mac} = \log \det \left(I + \mathbf{H}(S) Q_s \mathbf{H}^\dagger(S) (I + \mathbf{G}_s Q_p \mathbf{G}_s^\dagger)^{-1} \right) \quad (56)$$

Because for any positive definite matrix A and B , the function $\log \det(I + AB^{-1})$ is convex in B [18, Lemma II.3], we have

$$\mathcal{R}_{mac} = \mathbb{E}_{\mathbf{H}} [\mathbb{E}_{\mathbf{G}_s} [R_{mac} | \mathbf{H}]] \quad (57)$$

$$> \mathbb{E}_{\mathbf{H}} \left[\log \det \left(I + \mathbf{H}(S) Q_s \mathbf{H}^\dagger(S) (I + \mathbb{E}[\mathbf{G}_s Q_p \mathbf{G}_s^\dagger])^{-1} \right) \right] \quad (58)$$

$$= \mathbb{E}_{\mathbf{H}} \left[\log \det \left(I + \frac{\rho_s}{1 + P_p} \mathbf{H}(S) \mathbf{H}^\dagger(S) \right) \right] \quad (59)$$

where (58) uses the Jensen inequality and the fact that $\mathbf{H}(\mathcal{S})$ and \mathbf{G}_s are independent. Substituting Q_p from (8) and noting that $\mathbb{E}[\mathbf{G}_s \mathbf{G}_s^\dagger] = M I_{m \times m}$, we have (59).

Now we bound the right hand side of (59). Recall that $|\mathcal{A}|$ and $|\mathcal{S}|$ are the random number of eligible users and active users, respectively. By the Chebychev inequality, for any $\epsilon > 0$, we have

$$\mathbb{P}\left(|\mathcal{A}| > (1 - \epsilon)\bar{k}_s\right) > 1 - \frac{1 - p}{\epsilon^2 np} \quad (60)$$

$$= 1 - O(\bar{k}_s^{-1}) \quad (61)$$

where in the above we use the fact $\bar{k}_s = np$. Then, we expand (59) based the event $\{|\mathcal{A}| > (1 - \epsilon)\bar{k}_s\}$ and its complement, and discard the non-negative term associated with its complement:

$$\mathcal{R}_{mac} > \mathbb{E}\left[\log \det\left(I + \frac{\rho_s}{1 + P_p} \mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S})\right) \middle| |\mathcal{A}| > (1 - \epsilon)\bar{k}_s\right] \mathbb{P}\left(|\mathcal{A}| > (1 - \epsilon)\bar{k}_s\right) \quad (62)$$

$$\geq \mathbb{E}\left[\log \det\left(I + \frac{\rho_s}{1 + P_p} \mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S})\right) \middle| |\mathcal{A}| = (1 - \epsilon)\bar{k}_s\right] \left(1 - O(\bar{k}_s^{-1})\right) \quad (63)$$

$$= \mathbb{E}\left[\log \det\left(I + \frac{\rho_s}{1 + P_p} \mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S})\right) \middle| |\mathcal{S}| = (1 - \epsilon)\bar{k}_s\right] \left(1 - O(\bar{k}_s^{-1})\right) \quad (64)$$

where in the inequality (63), we apply the result in (61) and the fact that the conditional expectation of the right hand side of (62) is non-decreasing in $|\mathcal{A}|$. Since $|\mathcal{S}| = (1 - \epsilon)\bar{k}_s$ in case of $|\mathcal{A}| = (1 - \epsilon)\bar{k}_s$, then we obtain (64) due to the average throughput depending on $|\mathcal{A}|$ via the size of \mathcal{S} .

Recall that each entry of $\mathbf{H}(\mathcal{S})$ is i.i.d. $\mathcal{CN}(0, 1)$. Conditioned on $|\mathcal{S}| = (1 - \epsilon)\bar{k}_s$, $\mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S})$ is a Wishart Matrix with degrees of freedom $(1 - \epsilon)\bar{k}_s$, we have [19, Theorem 1]

$$\mathcal{R}_{mac} > \left(m \log\left(1 + \frac{(1 - \epsilon)\rho_s \bar{k}_s}{1 + P_p}\right) + O(\bar{k}_s^{-1})\right) \left(1 - O(\bar{k}_s^{-1})\right) \quad (65)$$

$$= m \log\left(1 + \frac{(1 - \epsilon)\rho_s \bar{k}_s}{1 + P_p}\right) + O\left(\frac{\log \bar{k}_s}{\bar{k}_s}\right) \quad (66)$$

$$= m \log \rho_s \bar{k}_s + m \log(1 - \epsilon) - m \log(1 + P_p) + O\left(\frac{\log \bar{k}_s}{\bar{k}_s}\right) \quad (67)$$

Since the above inequality holds for any $\epsilon > 0$, we have

$$\mathcal{R}_{mac} \geq m \log \rho_s \bar{k}_s - m \log(1 + P_p) + O\left(\frac{\log \bar{k}_s}{\bar{k}_s}\right) \quad (68)$$

Now we find an upper bound for \mathcal{R}_{mac} . For convenience, we denote

$$R_{mac,0} = \log \det\left(I + \rho_s \mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S}) + \mathbf{G}_s Q_p \mathbf{G}_s^\dagger\right) \quad (69)$$

and

$$R_I = \log \det\left(I + \mathbf{G}_s Q_p \mathbf{G}_s^\dagger\right) \quad (70)$$

So the average throughput can be written as

$$\mathcal{R}_{mac} = \mathbb{E}[R_{mac,0}] - \mathbb{E}[R_I] \quad (71)$$

Using the inequality $\det(A) \leq (\text{tr}(A)/k)^k$ [20], where A is a $k \times k$ positive definite matrix, $R_{mac,0}$ is bounded by

$$R_{mac,0} \leq m \log \left(1 + \frac{1}{m} \text{tr} \left(\rho_s \mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S}) + \mathbf{G}_s Q_p \mathbf{G}_s^\dagger \right) \right) \quad (72)$$

Therefore,

$$\mathbb{E}[R_{mac,0}] \leq m \mathbb{E} \left[\log \left(1 + \frac{1}{m} \text{tr} \left(\rho_s \mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S}) + \mathbf{G}_s Q_p \mathbf{G}_s^\dagger \right) \right) \right] \quad (73)$$

$$\leq m \log \left(1 + \frac{\rho_s}{m} \mathbb{E}[\text{tr}(\mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S}))] + \frac{1}{m} \mathbb{E}[\text{tr}(\mathbf{G}_s Q_p \mathbf{G}_s^\dagger)] \right) \quad (74)$$

$$\leq m \log (1 + \rho_s \bar{k}_s + P_p) \quad (75)$$

where (74) uses the Jensen inequality. To obtain the inequality (75), we use the facts that $\mathbb{E}[\text{tr}(\mathbf{G}_s Q_p \mathbf{G}_s^\dagger)] = P_p$ by substituting Q_p given by (8) as well as $\mathbb{E}[\text{tr}(\mathbf{H}(\mathcal{S}) \mathbf{H}^\dagger(\mathcal{S}))] \leq m \bar{k}_s$ due to $|\mathcal{S}| \leq \bar{k}_s$.

Now we lower bound the second term in (71). From [21, Theorem 1], we have

$$\mathbb{E}[R_I] \geq m_{\min} \log \left(1 + \frac{P_p}{M} \exp \left(\frac{1}{m_{\min}} \sum_{j=1}^{m_{\min}} \sum_{i=1}^{m_{\max}-j} \frac{1}{i} - \gamma \right) \right) \quad (76)$$

$$\triangleq \mathcal{R}_I \quad (77)$$

where $m_{\min} = \min(m, M)$, $m_{\max} = \max(m, M)$ and γ is the Euler's constant. Notice that \mathcal{R}_I is a finite constant independent of n and Γ .

Combining (75) and (77), we have

$$\mathcal{R}_{mac} \leq m \log(1 + \rho_s \bar{k}_s + P_p) - \mathcal{R}_I \quad (78)$$

Finally, substituting \bar{k}_s given by (20) and noting that $\bar{k}_s = \Theta(n^{\frac{1}{N+1}})$, we have

$$\mathcal{R}_{mac} \geq \frac{m}{N+1} \log n + \frac{1}{N+1} \log(\rho_s \Gamma^N) - m \log(1 + P_p) + O(n^{-\frac{1}{N+1}} \log n) \quad (79)$$

$$\mathcal{R}_{mac} \leq \frac{m}{N+1} \log n + \frac{1}{N+1} \log(\rho_s \Gamma^N) - \mathcal{R}_I + O(n^{-\frac{1}{N+1}}) \quad (80)$$

where we use the identity $\log(x + y) = \log x + \log(1 + x/y)$ in the above inequalities. This completes the proof. \square

APPENDIX B
PROOF OF THEOREM 3

Proof: We develop an upper bound for the secondary throughput in the presence of the primary broadcast only; the development is similar in the presence of the primary MAC and thus is omitted. We consider an arbitrary active user set \mathcal{S} and transmit covariance matrix given by (4), such that the interference constraints on the primary are satisfied.

By removing the interference from the primary to the secondary, the secondary throughput is enlarged. Then, using the inequality $\det(A) \leq (\text{tr}(A)/k)^k$ [20], where $A_{k \times k}$ is a positive definite matrix, we have

$$R_{mac} \leq m \log \left(1 + \frac{1}{m} \text{tr}(\mathbf{H}(\mathcal{S}) Q_s \mathbf{H}^\dagger(\mathcal{S})) \right) \quad (81)$$

Let \mathbf{h}_i be the $m \times 1$ vector of channel coefficients from the secondary user i ($i \in \mathcal{S}$) to the secondary base station, corresponding to a certain column of $\mathbf{H}(\mathcal{S})$. Since Q_s is diagonal, we have

$$\text{tr}(\mathbf{H}(\mathcal{S}) Q_s \mathbf{H}^\dagger(\mathcal{S})) = \sum_{i \in \mathcal{S}} \rho_i \text{tr}(\mathbf{h}_i \mathbf{h}_i^\dagger) \quad (82)$$

$$= \sum_{i \in \mathcal{S}} \rho_i |\mathbf{h}_i|^2 \quad (83)$$

$$\leq \max_{i \in \mathcal{S}} |\mathbf{h}_i|^2 \sum_{i \in \mathcal{S}} \rho_i \quad (84)$$

$$\leq \max_{1 \leq i \leq n} |\mathbf{h}_i|^2 \sum_{i \in \mathcal{S}} \rho_i \quad (85)$$

where ρ_i is the transmit power of the secondary user i . Let

$$P_{sum} = \sum_{i \in \mathcal{S}} \rho_i \quad (86)$$

and

$$h_{max} = \max_{1 \leq i \leq n} |\mathbf{h}_i|^2 \quad (87)$$

We can rewrite the right hand side of (81) as

$$R_{mac} \leq m \log \left(1 + \frac{1}{m} h_{max} P_{sum} \right) \quad (88)$$

We first bound P_{sum} and formulate an optimization as:

$$\begin{aligned} & \max_{\mathcal{S}, \{\rho_i\}} P_{sum} \\ & s.t. : \rho_i \leq \rho_s \text{ for } i \in \mathcal{S}, \\ & [\mathbf{G}_p Q_s \mathbf{G}_p^\dagger]_{\ell, \ell} \leq \Gamma \text{ for } 1 \leq \ell \leq N \end{aligned} \quad (89)$$

which is a standard linear programming, and the solution is denoted by P_{sum}^* . Then, P_{sum}^* is the maximum total transmit power, depending on the channel realizations for each transmission.

Subject to the interference constraints on the primary, the user selection and power allocation are coupled, and a direct analysis is difficult. Instead, we will find an upper bound for P_{sum}^* . Notice that the total interference (on all primary users) caused by the secondary user i is $\rho_i |\mathbf{g}_{p,i}|^2$, where $\mathbf{g}_{p,i}$ is the vector of channel coefficients from the secondary i to all N primary users. We relax the set of individual interference constraints in (89) with a single sum interference constraint:

$$\sum_{i \in \mathcal{S}} \rho_i |\mathbf{g}_{p,i}|^2 \leq N\Gamma \quad (90)$$

Notice that $\mathbf{g}_{p,i}$ corresponds to a certain column in \mathbf{G}_p .

Order the cross channel gains $\{|\mathbf{g}_{p,i}|^2\}_{i=1}^n$ of all the secondary users and denote the ordered cross channel gains by

$$|\tilde{\mathbf{g}}_{p,1}|^2 \leq |\tilde{\mathbf{g}}_{p,2}|^2 \leq \dots \leq |\tilde{\mathbf{g}}_{p,n}|^2 \quad (91)$$

Then, we further relax the sum interference constraint (90) by replacing $\{|\mathbf{g}_{p,i}|^2\}_{i \in \mathcal{S}}$ with the first $|\mathcal{S}|$ smallest cross channel gains $\{|\tilde{\mathbf{g}}_{p,i}|^2\}_{i=1}^{|\mathcal{S}|}$. Thus, we have:

$$\begin{aligned} & \max_{\mathcal{S}, \{\rho_i\}} P_{sum} \\ \text{s.t.: } & \sum_{i=1}^{|\mathcal{S}|} \rho_i |\tilde{\mathbf{g}}_{p,i}|^2 \leq N\Gamma \\ & \rho_i \leq \rho_s \text{ for } 1 \leq i \leq |\mathcal{S}| \end{aligned} \quad (92)$$

For any channel realizations, the solution for the above problem, denoted by $P_{sum,1}^*$, is always greater than, or equal to P_{sum}^* . Notice that $P_{sum,1}^*$ is also a random variable. Since $\{|\tilde{\mathbf{g}}_{p,i}|^2\}$ is in non-decreasing in i , the set of $\{\rho_i\}$ that achieves $P_{sum,1}^*$ satisfies $\rho_i \geq \rho_j$, for $i \leq j$. In other words, we have $\rho_i = \rho_s$, for $i = 1$ to $|\mathcal{S}| - 1$, and $\rho_i \leq \rho_s$, for $i = |\mathcal{S}|$.

Let S_{max} be the maximum value of $|\mathcal{S}|$ that satisfies the constraint

$$\rho_s \sum_{i=1}^{|\mathcal{S}|-1} |\tilde{\mathbf{g}}_{p,i}|^2 \leq N\Gamma \quad (93)$$

We have

$$P_{sum,1}^* \leq \rho_s S_{max} \quad (94)$$

where in (94) we have an inequality, because the constraint (93) is relaxed by discarding $\rho_{|\mathcal{S}|}$ compared to the interference constraint in (92).

Now, we focus on bounding $\rho_s S_{max}$. For any positive integer k , we have

$$\mathbb{P}(S_{max} < k) \geq \mathbb{P}\left(\sum_{i=1}^{k-1} |\tilde{\mathbf{g}}_{p,i}|^2 > \frac{N\Gamma}{\rho_s}\right) \quad (95)$$

which comes from the fact that the event of the right hand side implies the event of the left hand side. Notice that $\sum_{i=1}^{k-1} |\tilde{\mathbf{g}}_{p,i}|^2$ is a sum of least order statistics out of $\{|\mathbf{g}_{p,i}|^2\}_{i=1}^n$ with i.i.d. Gamma($N, 1$) distributions. We apply some results in the development of [13, Proposition 12], and obtain⁸

$$\mathbb{P}\left(\sum_{i=1}^{f(n)-1} |\tilde{\mathbf{g}}_{p,i}|^2 > \frac{N\Gamma}{\rho_s}\right) > 1 - O\left(\frac{1}{f(n)}\right) \quad (96)$$

where $f(n) = c_0 n^{\frac{1}{N+1}}$, and $c_0 = \left(\frac{\Gamma(N+1)}{(1-\epsilon)\rho_s} N^{-\frac{1}{N+1}}\right)^{\frac{N}{N+1}}$. For large N and small ϵ , $c_0 \approx \frac{\Gamma}{\rho_s}(N+1)$.

Let $k = f(n)$ in (95) and combine with (96):

$$\mathbb{P}\left(\rho_s S_{max} < \rho_s f(n)\right) > 1 - O\left(n^{-\frac{1}{N+1}}\right) \quad (97)$$

After characterizing $\rho_s S_{max}$, now we return to P_{sum}^* . To simplify notation, we denote

$$\bar{p}_{sum} = \rho_s f(n) \quad (98)$$

Because $P_{sum}^* \leq P_{sum,1}^* \leq \rho_s S_{max}$ for any channel realizations, from (97), we have

$$\begin{aligned} \mathbb{P}\left(P_{sum}^* \geq \bar{p}_{sum}\right) &= 1 - \mathbb{P}\left(P_{sum}^* < \bar{p}_{sum}\right) \\ &< 1 - \mathbb{P}\left(\rho_s S_{max} < \bar{p}_{sum}\right) \\ &< O\left(n^{-\frac{1}{N+1}}\right) \end{aligned} \quad (99)$$

Now, we complete the analysis of P_{sum}^* , and move to h_{max} . Because $\{|\mathbf{h}_i|^2\}_{i=1}^n$ have i.i.d. Gamma($m, 1$) distributions, using the similar arguments developed in Lemma 2, we obtain

$$\mathbb{P}\left(h_{max} > \zeta_n\right) = O\left(\frac{1}{\log n}\right) \quad (100)$$

$$\mathbb{E}[h_{max} \mid h_{max} > \zeta_n] < O(n \log n) \quad (101)$$

where ζ_n is a deterministic sequence satisfying

$$\zeta_n = \log n + m \log \log n + O(\log \log \log n) \quad (102)$$

⁸For our case, $\frac{1}{\lambda} = \gamma = N$.

Now we are ready to develop the upper bound for the secondary throughput. Since $P_{sum} \leq P_{sum}^*$, from (88), we have

$$\mathcal{R}_{mac} \leq m \mathbb{E}_{\mathbf{H}, P} \left[\log \left(1 + \frac{1}{m} h_{max} P_{sum}^* \right) \right] \quad (103)$$

$$\begin{aligned} &\leq m \mathbb{E}_{\mathbf{H}, P} \left[\log \left(1 + \frac{1}{m} h_{max} P_{sum}^* \right) \middle| P_{sum}^* < \bar{p}_{sum} \right] \mathbb{P}(P_{sum}^* < \bar{p}_{sum}) \\ &\quad + m \mathbb{E}_{\mathbf{H}, P} \left[\log \left(1 + \frac{1}{m} h_{max} P_{sum}^* \right) \middle| P_{sum}^* \geq \bar{p}_{sum} \right] \mathbb{P}(P_{sum}^* \geq \bar{p}_{sum}) \end{aligned} \quad (104)$$

$$\begin{aligned} &\leq m \mathbb{E}_{\mathbf{H}} \left[\log \left(1 + \frac{1}{m} h_{max} \bar{p}_{sum} \right) \right] \cdot 1 \\ &\quad + m \mathbb{E}_{\mathbf{H}} \left[\log \left(1 + \frac{1}{m} h_{max} \rho_s n \right) \right] \cdot O(n^{-\frac{1}{N+1}}) \end{aligned} \quad (105)$$

$$\begin{aligned} &\leq m \mathbb{E}_{\mathbf{H}} \left[\log \left(1 + \frac{1}{m} h_{max} \bar{p}_{sum} \right) \middle| h_{max} \leq \zeta_n \right] \mathbb{P}(h_{max} \leq \zeta_n) \\ &\quad + m \mathbb{E}_{\mathbf{H}} \left[\log \left(1 + \frac{1}{m} h_{max} \bar{p}_{sum} \right) \middle| h_{max} > \zeta_n \right] \mathbb{P}(h_{max} > \zeta_n) \\ &\quad + m \mathbb{E}_{\mathbf{H}} \left[\log \left(1 + \frac{1}{m} h_{max} \rho_s n \right) \middle| h_{max} \leq \zeta_n \right] \mathbb{P}(h_{max} \leq \zeta_n) O(n^{-\frac{1}{N+1}}) \\ &\quad + m \mathbb{E}_{\mathbf{H}} \left[\log \left(1 + \frac{1}{m} h_{max} \rho_s n \right) \middle| h_{max} > \zeta_n \right] \mathbb{P}(h_{max} > \zeta_n) O(n^{-\frac{1}{N+1}}) \end{aligned} \quad (106)$$

$$\begin{aligned} &\leq m \log \left(1 + \frac{1}{m} \zeta_n \bar{p}_{sum} \right) \cdot 1 \\ &\quad + m \log \left(1 + \frac{\bar{p}_{sum}}{m} \mathbb{E}[h_{max} | h_{max} > \zeta_n] \right) \mathbb{P}(h_{max} > \zeta_n) \\ &\quad + m \log \left(1 + \frac{1}{m} \zeta_n \rho_s n \right) \cdot 1 \cdot O(n^{-\frac{1}{N+1}}) \\ &\quad + m \log \left(1 + \frac{\rho_s n}{m} \mathbb{E}[h_{max} | h_{max} > \zeta_n] \right) \mathbb{P}(h_{max} > \zeta_n) O(n^{-\frac{1}{N+1}}) \end{aligned} \quad (107)$$

$$\begin{aligned} &\leq m \log \left(1 + \frac{1}{m} \zeta_n \bar{p}_{sum} \right) \\ &\quad + m \log \left(1 + \frac{\bar{p}_{sum}}{m} O(n \log n) \right) O\left(\frac{1}{\log n}\right) \\ &\quad + m \log \left(1 + \frac{1}{m} \zeta_n \rho_s n \right) O(n^{-\frac{1}{N+1}}) \\ &\quad + m \log \left(1 + \frac{\rho_s n}{m} O(n \log n) \right) O\left(\frac{1}{\log n}\right) O(n^{-\frac{1}{N+1}}) \end{aligned} \quad (108)$$

where the second term in (105) comes from using (99) as well as the fact that P_{sum}^* is upper bounded by $\rho_s n$. In (107), we apply the Jensen inequality to obtain the second and fourth terms. Using (100)

and (101), we have the second and fourth terms in (108). Finally, by substituting \bar{p}_{sum} and ζ_n , we obtain

$$\mathcal{R}_{mac} \leq \frac{m}{N+1} \log n + O(\log \log n) \quad (109)$$

This concludes the proof of this theorem. \square

APPENDIX C

PROOF OF LEMMA 2

Proof: First, we prove (49). Let $Z = |\mathbf{h}_i^\dagger \phi_j|^2$ and $Y = \theta(\sum_{k \neq j} |\mathbf{h}_i^\dagger \phi_k|^2 + |\mathbf{g}_{s,i}|^2)$. Then, Z has the exponential distribution, and Y has the Gamma($(m+M-1), \theta$) distribution. We can write

$$L_i = \frac{Z}{c+Y} \quad (110)$$

where $c = \frac{m}{\rho}$. Conditioned on Y , the pdf of L_i is given by

$$f_L(x) = \int_0^\infty f_{L|Y}(x|y) f_Y(y) dy \quad (111)$$

$$= \int_0^\infty (c+y) e^{-(c+y)x} \times \frac{y^{m+M-1} e^{-y/\theta}}{(m+M-1)! \theta^{m+M}} dy \quad (112)$$

$$= \frac{e^{-cx}}{(1+\theta x)^{m+M}} (c(1+\theta x) + \theta(m+M-1)) \quad (113)$$

So the cdf of L_i is

$$F_L(x) = 1 - \int_x^\infty f_L(t) dt \quad (114)$$

$$= 1 - \frac{e^{-cx}}{(1+\theta x)^{m+M-1}} \quad (115)$$

We define a grow function as

$$g_L(x) = \frac{1 - F_L(x)}{f_L(x)} \quad (116)$$

$$= \frac{1 + \theta x}{c(1 + \theta x) + \theta(m + M - 1)} \quad (117)$$

Since $\lim_{x \rightarrow \infty} g'_L(x) = 0$, the limiting distribution of $L_{max} = \max_{1 \leq i \leq n} L_i$ exists [22]:

$$\lim_{n \rightarrow \infty} (F_L(b_n + a_n x))^n = e^{-e^{-x}} \quad (118)$$

where $b_n = F_L^{-1}(1 - 1/n)$ and $a_n = g_L(b_n)$. In general, an exact closed-form solution for a_n and b_n is intractable, but an approximation can be obtained, which is sufficient for asymptotic analysis. After manipulating (115), we have

$$b_n = \frac{1}{c} \log n - \frac{m+M-1}{c} \log \log n + O(\log \log \log n) \quad (119)$$

and thus

$$a_n = \frac{1}{c} + O\left(\frac{1}{\log n}\right) \quad (120)$$

It is straightforward to verify $\lim_{n \rightarrow \infty} (ng'_L(b_n)) = \infty$, so we apply the expansion developed in [23, Eq. (22)]

$$(F_L(b_n + a_n x))^n = \exp\left(-\exp(-x + \Theta(\frac{x^2}{\log^2 n}))\right) \quad (121)$$

Let $x_1 = -\log \log n$ and substitute x_1 into (121), we obtain (49).

Now, we prove (50) and (51). Since U_i is similar to L_i , except that the denominator now has the Gamma(M, θ) distribution. Following the same steps of obtaining (121), we have the expansion of the cdf of U_{max} :

$$(F_U(d_n + c_n x))^n = \exp\left(-\exp(-x + \Theta(\frac{x^2}{\log^2 n}))\right) \quad (122)$$

where

$$d_n = \frac{1}{c} \log n - \frac{M}{c} \log \log n + O(\log \log \log n) \quad (123)$$

and

$$c_n = \frac{1}{c} + O\left(\frac{1}{\log n}\right) \quad (124)$$

(50) follows by substituting $x_2 = \log \log n$ into (122).

Finally, because $\mathbb{E}[U_{max}] < n\mathbb{E}[U_i]$ [22], we have

$$\mathbb{E}\left[U_{max} \mid U_{max} > d_n + \frac{1}{c} \log \log n\right] \leq \frac{n\mathbb{E}[U_i]}{\mathbb{P}(U_{max} > d_n + \frac{1}{c} \log \log n)} \quad (125)$$

$$= \Theta(n \log n) \quad (126)$$

where we use (50) in the last equality. \square

APPENDIX D

PROOF OF THEOREM 4

Proof: We first find a lower bound for the secondary average throughput \mathcal{R}_{bc} . We condition on $P = \rho$ and let $l_n = b_n - \frac{\rho}{m} \log \log n$, where b_n is given by Lemma 2. Using (48) and Lemma 1, the conditional throughput $\mathcal{R}_{bc|P}(\rho)$ can be bounded as

$$\mathcal{R}_{bc|P}(\rho) \geq m\mathbb{E}\left[\log(1 + L_{max}) \mid P = \rho\right] \quad (127)$$

$$\geq m\mathbb{E}\left[\log(1 + L_{max}) \mid L_{max} \geq l_n, P = \rho\right] \mathbb{P}(L_{max} \geq l_n \mid P = \rho) \quad (128)$$

$$> m \left(\log \left(\frac{\rho}{m} \log n \right) + O \left(\frac{\log \log n}{\log n} \right) \right) \left(1 - \Theta(n^{-1}) \right) \quad (129)$$

$$= m \log \left(\frac{\rho}{m} \log n \right) + O \left(\frac{\log \log n}{\log n} \right) \quad (130)$$

From (127) to (128), we discard the non-negative term associated with the event $\{L_{max} < l_n\}$. Using (49) from Lemma 2 and the identity $\log(x + y) = \log x + \log(1 + y/x)$, we have (129).

Now we take the expectation with respect to P . From (39), we have

$$P > \frac{m\Gamma}{\max_{1 \leq i \leq N} |\mathbf{g}_{p,i}^\dagger|^2 + m\Gamma/P_s} \quad (131)$$

where $\mathbf{g}_{p,i}^\dagger$ is the $1 \times m$ vector of channel coefficients from the secondary base station to the primary user i . Let the pdf of $\max_{1 \leq i \leq N} |\mathbf{g}_p(i)|^2$ be $f_{g_p}(x)$. Because the random variable P is (stochastically) greater than the right hand side of (131), from Lemma 1 and (130), we have

$$\mathcal{R}_{bc} > \int_0^\infty m \log \left(\frac{\Gamma \log n}{x + m\Gamma/P_s} \right) f_{g_p}(x) dx + O \left(\frac{\log \log n}{\log n} \right) \quad (132)$$

$$\geq m \log \left(\frac{\Gamma \log n}{\tilde{\mu}_1 + m\Gamma/P_s} \right) + O \left(\frac{\log \log n}{\log n} \right) \quad (133)$$

$$= m \log (\Gamma \log n) - m \log (\tilde{\mu}_1 + m\Gamma/P_s) + O \left(\frac{\log \log n}{\log n} \right) \quad (134)$$

where (133) comes from the convexity of $\log(a + \frac{b}{x+c})$ and

$$\tilde{\mu}_1 = \mathbb{E}[\max_{1 \leq i \leq N} |\mathbf{g}_p(i)|^2] \quad (135)$$

To find an upper bound, we still begin with the conditional throughput $\mathcal{R}_{bc|P}(\rho)$. Let $u_n = d_n + \frac{\rho}{m} \log \log n$, where d_n is given by Lemma 2. Then

$$\mathcal{R}_{bc|P}(\rho) \leq m \mathbb{E} \left[\log(1 + U_{max}) \mid P = \rho \right] \quad (136)$$

$$\leq m \mathbb{E} \left[\log(1 + U_{max}) \mid U_{max} < u_n, P = \rho \right] \mathbb{P}(U_{max} < u_n \mid P = \rho) \quad (137)$$

$$+ m \mathbb{E} \left[\log(1 + U_{max}) \mid U_{max} \geq u_n, P = \rho \right] \mathbb{P}(U_{max} \geq u_n \mid P = \rho) \quad (138)$$

$$< m \log(1 + u_n) \left(1 - \Theta \left(\frac{1}{\log n} \right) \right) + m \log \left(1 + \mathbb{E}[U_{max} \mid U_{max} \geq u_n, P = \rho] \right) \Theta \left(\frac{1}{\log n} \right) \quad (139)$$

$$< m \log \left(1 + \frac{\rho}{m} \log n \right) + O(1) \quad (140)$$

where (136) comes from (48). We apply (50) in Lemma 2 and the Jensen inequality to obtain (139). Using (51) in Lemma 2 and substituting u_n , we obtain (140).

After calculating an upper bound for the conditional throughput, we average over P . From (39), we have

$$P \leq \frac{m\Gamma}{\max_{1 \leq i \leq N} |\mathbf{g}_{p,i}^\dagger|^2} \quad (141)$$

We denote

$$\frac{1}{\tilde{\mu}_2} = \mathbb{E}\left[1 / \max_{1 \leq i \leq N} |\mathbf{g}_{p,i}^\dagger|^2\right] \quad (142)$$

Then, by the Jensen inequality, we have

$$\mathcal{R}_{bc} < m \log \left(1 + \frac{\log n}{m} \mathbb{E}[P]\right) + O(1) \quad (143)$$

$$< m \log \left(1 + \frac{\Gamma}{\tilde{\mu}_2} \log n\right) + O(1) \quad (144)$$

$$= m \log(\Gamma \log n) - m \log \tilde{\mu}_2 + O(1) \quad (145)$$

where (144) holds since $\mathbb{E}[P] \leq \frac{m\Gamma}{\tilde{\mu}_2}$. The theorem follows. \square

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